

Set 2 Problems 12 October 2009

Sixth Form:

1) A draw contains an unsorted collection of black, navy, white, and brown coloured socks. If socks are taken at random, one at a time, what is the minimum number which must be taken to be certain of finding five matching pairs?

2) Given that n is a natural number, when is $n^4 + 4$ prime?

3) Consider the function, $f(x) = x + 1/x$.

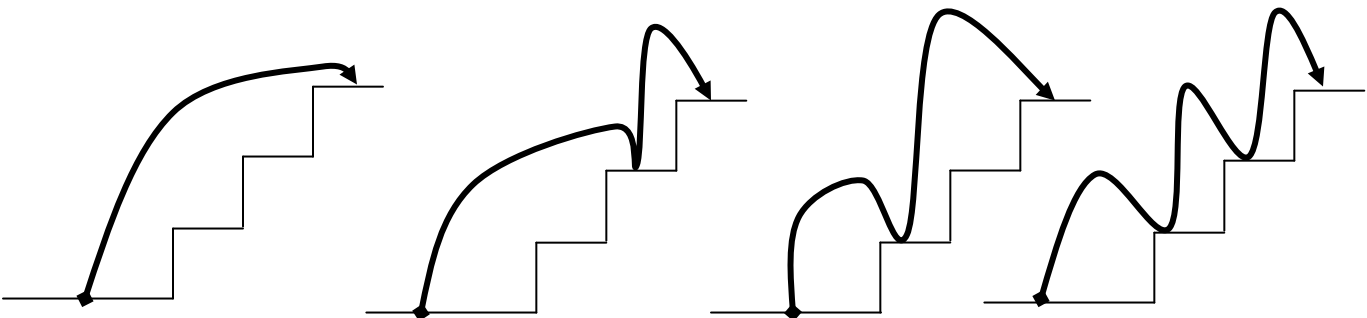
$$f(2) = 2 + 1/2 = 2.5$$

$$f(0.4) = 0.4 + 1/0.4 = 2.9$$

$$f(1.25) = 1.25 + 1/1.25 = 2.05$$

Prove that $f(x) \geq 2$ for all real values of $x \neq 0$.

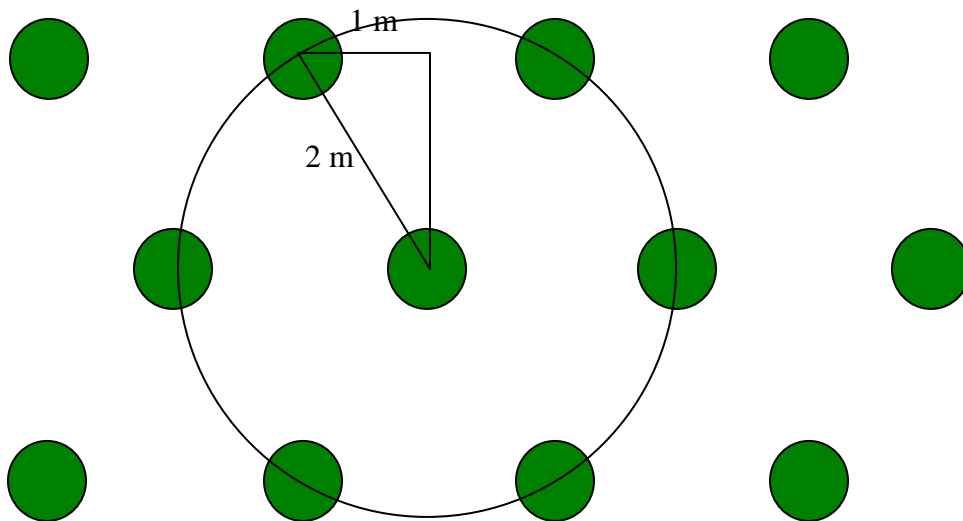
4) It is possible to climb three steps in exactly four different ways.
How many ways can you climb ten steps?



Year 10 /11:

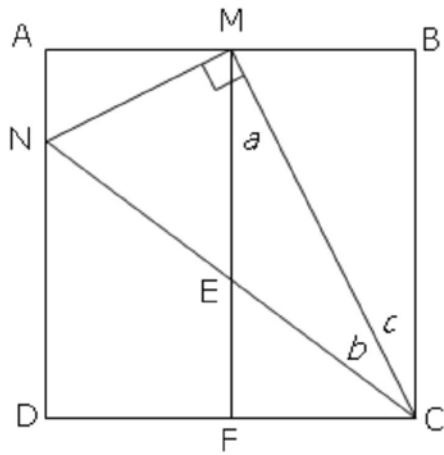
1) Given that $x^2 + x + 1 = 0$, find the value of x^3 .

2) When Christmas trees are planted they should stand at least 2 metres away from one another whilst growing.



What is the maximum number of trees that can be planted in one square kilometre?

3) In the square ABCD, M is the midpoint of AB.
A line is drawn through M perpendicular to CM to locate N.

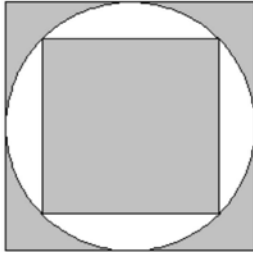


Prove that the size of angle BCM is equal to the size of angle MCN.

Year 9:

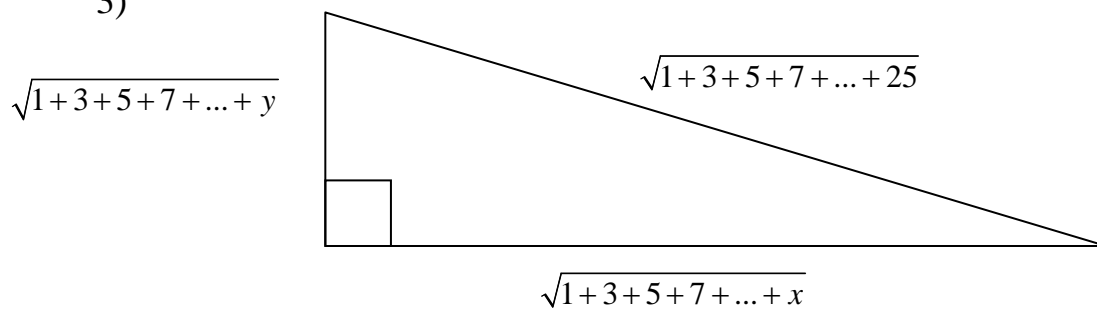
1) Without the use of a calculator determine which is greater in value, the square root of two or the cube root of three.

2) A grey square has a white circle inscribed and a grey square inscribed in the circle.



What fraction of the diagram is white?

3)



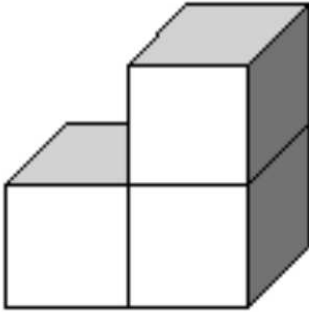
The length of the hypotenuse of a right-angled triangle is $\sqrt{1+3+5+7+\dots+25}$

The lengths of the other two sides are given by $\sqrt{1+3+5+7+\dots+y}$

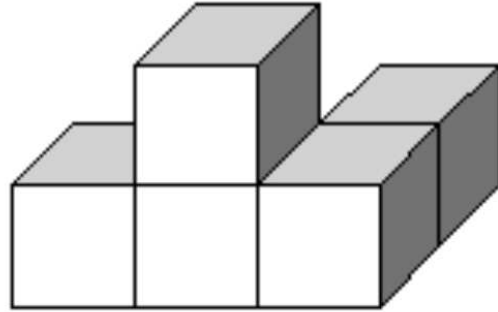
and $\sqrt{1+3+5+7+\dots+x}$ where x and y are positive integers. What is the value of $x + y$?

Year 7 /8

1) In box world all the buildings are made from cubic box sections.

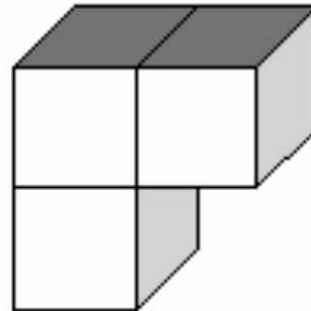
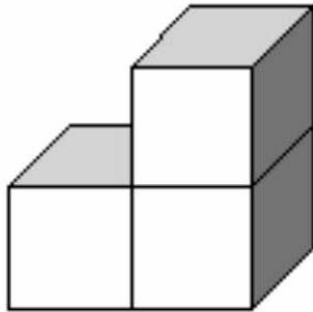


3 rooms



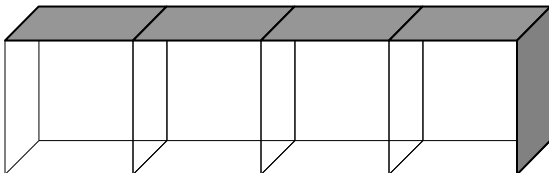
5 rooms

Not counting different orientations, (so the 2 diagrams below are the same)

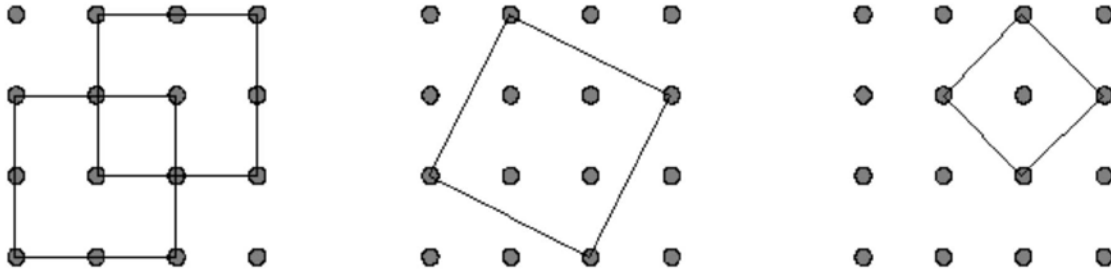


how many distinct ways can you connect four rooms together?

One obvious way is shown below



2) By joining dots together, how many squares can you draw on a grid measuring 4 dots by 4 dots?



3) A group of Archaeologists discovered some simple hieroglyphs on the stone lid of a tomb in Egypt. When they translated them they realised that it was a four digit number, but more remarkably it is the smallest number that can be divided by all of the numbers from 1 to 10 without any remainder. What was that number?

4) The end of topic tests in mathematics are looming and you realise that you have spent one too many lunch times surfing the internet and not revising. Just before you log off you suddenly remember that you never actually logged into the terminal. Curious about who left the computer logged in, you discover that it was your mathematics teacher.

You decide to "research" the contents of his private directory and stumble upon a file called, "end_of_topic_test.doc". After checking that no one is looking, you double click on the filename and are greeted with a box requesting a password. Not to be out-done, you look through the files and discover a file called, "password.gif". You cannot believe your luck! Excitedly you double click it and are presented with the following image:

ΑΡΧΙΜΗΔΗΣ

Can you discover the password?
(You may need to do a bit of research on the internet)

Open to All Years (Hint: You will need to know about binary numbers)

At school, Bill Speareshake was always the class joke, but in recent years he has met great success as a poet and playwright. Many people believe that his equally recent involvement with people in high places is a mere coincidence, but others suspect that the real genius behind his works is none other than the esoteric figure Crispin Bacon. In fact, the skeptics believe that his latest masterpiece, in the form of a short poem, contains the identity of the true author.

a doZen, A grOsS, And a SCorE
pLUS 3 TIMES ThE squArE RooT of 4
DiViDED by 7
PLUS 5 Times 11
is 9 sqUareD, AnD NOT a BIT more
Can you decode the message solve the mystery?